2019
Team Round

Name: $\qquad$ Score: $\qquad$ / 160

PLEASE DO NOT FILL IN ABOVE! (the SCORE blank)
Grade: $\qquad$ Team: $\qquad$

This is a round consisting of 10 challenging problems to be done in 30 minutes. You may communicate and discuss problems with people on your team. Problems are in roughly ascending difficulty, and each problem is worth 6 points. Any figures in the test may not be to scale.

No aids are permitted aside from pencils, pens, and provided scratch paper. In particular, no calculators or other computers are permitted. Communication with other people on your own team is allowed.

Record your answers in the box corresponding to the correct problem. Only answers printed in the boxes below will be scored.

## Your Answers

|  | 3. | 5. | 7. | 9. |
| :--- | :--- | :--- | :--- | :--- |
| 2. | 4. | 6. | 8. | 10. |

1. Given the fraction $\frac{1280}{500}$, what's the smallest positive number $n$ such that $\frac{1280}{500} \cdot n$ is a whole number?
2. Three security cameras act as surveillance for the local bank. The first camera takes a photo every 2 minutes, the second camera takes a photo every 5 minutes, and the third camera takes a photo every 10 minutes. However, it is unknown when they were first turned on. A thief arrives at the bank at midnight (12:00 AM) and escapes at exactly 12:01 AM. What is the probability that at least one of the security cameras managed to get a photo of the thief?
3. The Winchester Paint company makes three shades of green paint: light-green, regular-green, and darkgreen. Light-green is made from $20 \%$ blue paint and $80 \%$ yellow paint. Regular-green is made from $50 \%$ light-green paint and $50 \%$ blue paint. Dark-green is made from $60 \%$ regular-green paint and $40 \%$ blue paint. To make an equal amount of these three green paints, what is the ratio of blue to yellow paints needed? Express your answer as a common fraction.
4. Find the value of $\frac{123456789}{9}+\frac{234567891}{9}+\ldots+\frac{891234567}{9}+\frac{912345678}{9}$.
5. Given a sequence of numbers, a valid move consists of swapping two digits in a way such that the digit on the left is greater before the swap (e.g. given the number 2019, you can swap the digits 2 and 1 , but you cannot swap 0 and 1 ). What is the greatest number of moves that can be performed on the number 987654321 ?
6. Find all integers $n$ such that $n^{2}+20 n+19$ is prime.
7. In the figure below, $A B C$ is a right triangle with $A B=8$ and $B C=6$. Point $D$ lies on $B C$ such that $C D=3$, and point $E$ is a point such that $\overline{D E}$ is parallel to $\overline{A C}$ and $\angle A E C=90^{\circ}$. What is the value of $A E \times E C \times C A$ ?

8. A group of animals is split into three teams. Team $A$ has 3 dog and 4 cats, team $B$ has 4 dogs and 3 cats, and team $C$ has 5 dogs and 2 cats. In a competition with 120 mice, team $A$ caught $15 \%$ more mice than team $B$. Assuming that each animal of the same type catches mice at a constant rate, how many mice did team $C$ catch?
9. There exists a $5 \times 5$ grid made out of squares. Chad wants to fill up 20 squares in the grid in a way such that none of the rows, columns, nor diagonals (a diagonal has 5 squares) are completely filled. How many different ways can Chad do this?
10. In $\triangle A B C$, points $D$ and $E$ are on $\overline{A B}$ and $\overline{A C}$ respectively such that $A D=20$ and $B D=19$. If $\mathrm{m} \angle A B E=2 \mathrm{~m} \angle B A E$, find the length of $B E$ that will minimize the perimeter of $\triangle B D E$.

