Set 1

1. **(4)** Simplify: $\frac{1}{2} \times \frac{4}{5} \times \frac{10}{11} \times \frac{22}{23}$.

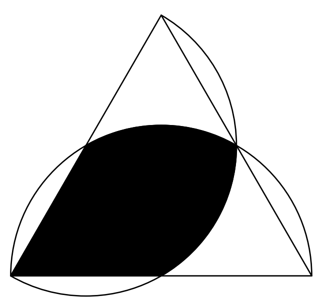
2. **(4)** Two squares, A and B, have side lengths 10 and 25 respectively. How many times the area of square A is the area of square B?

3. **(4)** a, b, and c are unique even integers. If $a + b + c = 2019$, find the number of possible values of a.

Team: ________________

Set 2

4. **(5)** The figure below is made up of an equilateral triangle and two semicircles. What fraction of the entire figure is shaded?



5. **(5)** If 18 apples are worth 32 bananas and the cost of apples and bananas are both an integer number of dollars, then what is the smallest possible cost of one banana?

6. **(5)** Two points A and B are chosen on the circumference of a semicircle centered at O. What is the probability that $\angle AOB$ is an obtuse angle?

Team: ________________

Set 3

7. **(6)** Evaluate $99^2 + 2 \times 99$.

8. **(6)** Consider a 5 by 5 board of tiles. If Josh has a hammer that breaks a tile and tiles adjacent to it (sharing an edge), what is the minimum amount of hammer swings required to break all 25 tiles?

9. **(6)** Brandon rolls two fair six-sided die. What is the probability that the product of the two numbers is a perfect square?

Team: ____________________

Set 4

10. **(7)** Brandon is in a chess tournament with a double elimination knockout bracket (You have to lose two matches to be out of the tournament. After your first loss, you go to the losers bracket and play there). If there are 128 players at the beginning, how many games will be played in this tournament assuming everyone loses at least once?

11. **(7)** Let $ABCD$ be a square with side length 5. The set of points outside the square with distance exactly 1 from the square's border traces a curve with perimeter p. Find p.

12. **(7)** How many ordered quintuples of integers (a, b, c, d, e) satisfy the equation

$$(a-1)^2 + (b-2)^2 + (c-3)^2 + (d-4)^2 + (e-5)^2 = 3?$$

Team: ____________________

Set 5

13. **(8)** How many perfect squares are greater than 6^4 but less than 4^6?

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14. **(8)** Alex writes down all the perfect squares from 1 to 900. How many digits did he write?

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15. **(8)** Brandon lists out all numbers from 1 through 10. He then randomly chooses 2 not necessarily distinct numbers from the list (Ex: He can pick the number 3 twice). If the pairing has never been drawn before, he sums the two numbers and records them on a piece of paper. He stops after he has chosen all possible pair combinations. What is the sum of all the numbers recorded on the piece of paper?

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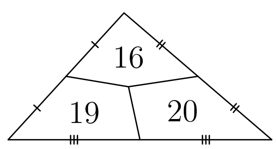
Team: ______________________

Set 6

16. **(9)** In a triangle, a point is drawn such that it is equidistant from all three sides. The triangle is split into three quadrilaterals with areas 16, 19, and 20 by constructing a line segment this point to the midpoint of each side. If the side lengths of the triangle form a simplified ratio of $a : b : c$ where $a \leq b \leq c$, find (a, b, c).



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17. **(9)** For x and y such that $x^2 + y^2 + 20xy = 19$. Find the sum of all positive values of xy for which $x + y$ is a natural number.

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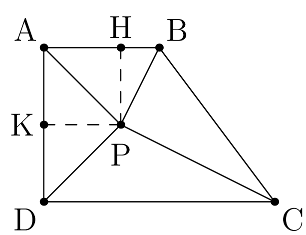
18. **(9)** A plot of land is in the shape of a right trapezoid as shown below, with $\overline{AB} = 27$m, $\overline{BC} = 45$m, $\overline{CD} = 54$m and $\overline{AD} = 36$m. A point P is chosen inside to divide the land into four smaller and equally sized plots of land. If H and K are the foot of the altitudes from P to $\overline{AB}$ and $\overline{AD}$ respectively, find the area of $AHPK$ in meters squared.



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Team: ______________________

Set 7

19. **(10)** For a positive integer n, $201n^9$ has exactly 1210 factors, including 1 and itself. What is the smallest possible value of n?

20. **(10)** Given $a+b=10$, $c+d=39$, and $(a-c)(b-d)=190$. Find all possible values of $a+d$.

21. **(10)** Find the area of a right triangle with an altitude to the hypotenuse of length 1 and a perimeter of length 9.

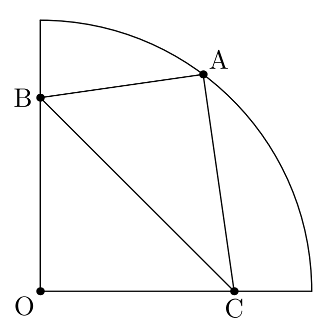
Team: ________________

Set 8

22. **(11)** Farmer Wilson owns a farm divided into 342 identical square fields by a 18×19 grid. Each field is either used for irrigation or planting crops. Due to the hot and dry climate, each irrigation field can only support up to two adjacent crop fields. What is the maximum number of fields which Wilson can plant?

23. **(11)** In a game, the nth win in a win streak award the player $2n+1$ points while the nth loss in a losing streak deducts $n+1$ points. For example, the third win in a win streak awards the player 7 points. If Kevin suffered 48 losses in a row, how many consecutive wins must he achieve to earn back the same number of points which he has lost?

24. **(11)** $\triangle ABC$ with $AB=12$, $AC=16$, and $BC=20$ is inscribed inside a quarter circle centered at O. If $\overline{AO}$ bisects $\angle A$, find the area of the quarter circle.



Team: ________________