WinMaC 2018

Team Round

Name: ______________________________ Score: ____________ / 160

PLEASE DO NOT FILL IN ABOVE! (the SCORE blank)

Grade: ____________________ Team: ____________________

This is a round consisting of 10 challenging problems to be done in 30 minutes. You may communicate and discuss problems with people on your team. Problems are in roughly ascending difficulty, and each problem is worth 6 points. Any figures in the test may not be to scale.

No aids are permitted aside from pencils, pens, and provided scratch paper. In particular, no calculators or other computers are permitted. Communication with other people on your own team is allowed.

Record your answers in the box corresponding to the correct problem. Only answers printed in the boxes below will be scored.

Your Answers

1.	3.	5.	7.	9.
2.	4.	6.	8.	10.

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1. What is the value of

$$(1+2)-(2+3)+(3+4)-\ldots-(2016+2017)+(2017+2018)?$$

2. Robert and Jason are working on WinMaC questions. It takes Robert 7 minutes to write a WinMaC question and Jason 3 minutes to solve the question Robert wrote. If they work continuously for 45 minutes, what fraction of that time is Jason waiting for Robert to finish writing his question?

3. Find the positive difference between the smallest and largest four digit multiples of 18 which end in 18 and have a digit sum of 18.

4. Given a quadrilateral $ABCD$ with a right angle at B, satisfying $AB = 12$, $CD = 13$, and $BC = 5$. Furthermore, let the diagonal $\overline{AC}$ cut the angle $\angle BCD$ into two parts with equal measure. If point E is chosen on $\overline{AC}$ such that $AE = 3$, then what is the area of $\triangle ADE$?

5. Given that n is a natural number, and the sum of the digits of $3n$ is divisible by 11, then what is the minimum possible value of n?

6. Let x be a natural number. If $2x$ is the result of doubling each digit in x, consider $2x$ to be a *doubled* number. How many three-digit *doubled* numbers are there?

7. The factorial of a positive integer n, denoted $n!$, is defined as $n \cdot (n-1) \cdot (n-2) \cdots 2 \cdot 1$. Given that

$$(x-2)(x-4)! = (x-2)!(x-3),$$

solve for all possible values of x.

8. Nine people are seated in a row. Every person, except the leftmost one, says, "The person directly to my left has black hair." Exactly half of their statements are true (and the other half are false). How many combinations of people in this row could have black hair?

9. Find the sum of all positive integers x such that $(x-1)+(\frac{x-2}{2})^2+(\frac{x-3}{3})^3+(\frac{x-4}{4})^4$ is an integer less than 2018.

10. P is a point inside parallelogram $ABCD$ and H is the foot of the perpendicular from P to $\overline{AB}$. $AD = AP$ and $DP = 2HP$. If $\angle A \cong \angle DPH$, find $m\angle B$.