## WinMaC 2016

## Team Round

Name: $\qquad$ Score: $\qquad$ / 60

## PLEASE DO NOT FILL IN ABOVE! (the SCORE blank)

Grade: $\qquad$ Team: $\qquad$

This is a round consisting of 20 problems to be done in 45 minutes. You may communicate and discuss problems with people on your team. Problems are in roughly ascending difficulty, and each problem is worth 6 points. Any figures in the test may not be to scale.

There is a challenging 4 problem bonus section, where for each problem you are correct on, you earn an extra 20 points. These problems are much harder than the other team problems!

No aids are permitted aside from pencils, pens, and provided scratch paper. In particular, no calculators or other computers are permitted. Communication with other people on your own team is allowed.

Record your answers in the box corresponding to the correct problem. Only answers printed in the boxes below will be scored.

## Your Answers

| 1. | 6. | 11. | 16. |
| :--- | :--- | :--- | :--- |
| 2. | 7. | 12. | 17. |
| 3. | 8. | 13. | 18. |
| 4. | 9. | 14. | 19. |
| 5. | Bonus \#2: | Bonus \#3: | Bonus \#4: |
| Bonus \#1: |  | 15. |  |

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1. Lag is measured in milliseconds by game players, it can be calculated by the formula $\frac{p^{2}-900}{360}$ where $p$ stands for ping. How many milliseconds does a player spend in lag if their ping is $90 ?$
2. If 2 cheeseburgers costs $\$ 5$ and a slice of cheese costs $\$ 0.50$, how many dollars would a hamburger (cheeseburger with no cheese) cost if a cheeseburger only has one slice of cheese in it?
3. A 2 foot tall Squirtle can shoot water a distance of 30 feet. Wartortle can shoot the water 75 feet, and he is 5 feet tall. Blastoise is 10 feet tall. Assuming that these shell pokemon can shoot water proportional to their height, how far can Blastoise shoot water, in feet?
4. There are 37 sanic mice in a cage. If they have either 4 legs or 5 legs and there is a total of 173 legs in the cage, then how many sanic mice have 5 legs?
5. Jeff has a bag of 10 nuts, there are 3 peanuts, 5 cashews, and 2 almonds. Jessica wants to make cashew-peanut butter, which requires 2 peanuts and 1 cashew. If she randomly grabs 3 nuts out of Jeffs bag, what is the probability that the nuts she grabbed out of the bag can make cashew-peanut butter?
6. Robert was holding onto a balloon, before the balloon slipped away and began to ascend into the sky. The balloon ascends away from Robert at a rate of 5 feet per minute, and Robert began to panic. If Robert can reach things at most 12 feet higher than him, in how many seconds will the balloon be unreachable?
7. How many 6 digit numbers end in 2016 ?
8. A very ancient language known as Zijian had only 4 letters in it, which are $w, n$, i, and $u$. How many different combinations of 5 letter words are possible in Zijian (the words do not have to exist in English)?
9. Recently, a new type of cat nicknamed "Eo" has been discovered to make sanic mice to behave insanically. Studies have shown that a group of $n$ sanic mice need 2 more "Eo"s than a group of $n-1$ sanic mice in order to behave insanically. If a group of 20 sanic mice require 2016 "Eo"s to make them insanic, then how many "Eo"s are needed to make 37 sanic mice behave insanically?

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10. If $f(x)=3 x+1$, and $g(x)=(x-1) / 3$, then what is $f(g(f(g(f(g(g(4)))))))$ ?
11. There exist a right triangle of integer side lengths such that the sum of the squares of each side is 50 . What is the area of this triangle?
12. What is 5 more than 6 times the answer to this problem?
13. According to the weather forecast, there is a $40 \%$ chance it will rain on Tuesday and a $30 \%$ chance that it will rain on Wednesday if it doesn't rain on Tuesday. However, if it rains on Tuesday, then the chance that it will rain on Wednesday will rise to $60 \%$. What is the probability that it will rain on Wednesday? Express your answer as a percent.
14. There exists a positive integer $n$ such that both $n+11$ and $n+22$ are perfect squares. Compute $n$.
15. Kaiwen has to type up an infinitely long essay for his English teacher. Kaiwen starts typing at a rate of 45 words per minute, starting at 4:00 PM. Unfortunately, Will Sun decided to start deleting the words that Kaiwen typed at a rate of 50 words per minute. What time should Will Sun start deleting words to delete Kaiwens entire essay at exactly 7:20 PM, assuming that from the time he starts deleting to 7:20 he is deleting and Kaiwen is typing words the whole time?
16. $\triangle A B C$ is a right triangle with $\angle A=90^{\circ}$ and legs equal to integers $x$ and $y$. If the hypotenuse has length 65 , and $x>y$, list all ordered pairs $(x, y)$. (The order of your list doesnt matter)
17. $x^{4}+4=\left(x^{2}+a x+b\right)\left(x^{2}+c x+d\right)$ for integers $a, b, c, d$. Determine the ordered quadruple $(a, b, c, d)$.
18. Let $f(x)=x^{2}+x+\frac{1}{3}$ for reals $x$. Determine the value of $f(1)+f(2)+f(3)+f(4)+\cdots+f(19)$.
19. There exist a fraction such that it is $\frac{2}{7}$ when simplified, but when the integer $X$ is subtracted from both the numerator and denominator, it is simplified to $\frac{5}{18}$. What is the smallest possible value of $X$ ?
20. Circles $S, Q$, and $R$ are inscribed in equilateral $\triangle A B C$, where the three circles are mutually tangent and have the same areas. $M$ and $N$ are the midpoints of $\overline{A B}$ and $\overline{B C}$ respectively. Given that $A B=1$, find the area of $M Q N R S$.

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## Bonus Section

1. Jason is a terrible problem writer. He has written 4 different terrible problems which he brings to McCall and presents to Robert Zhang, Will Niu, Will Sun, and Will Huang. Each of Robert, Will N. , Will S., and Will H. randomly choose exactly one of Jason's problems to laugh at. What is the expected number of questions that dont get laughed at by someone with the first name of Will? (A problem can possibly have multiple people laughing at it, or no one laughing at it.)
2. A rectangle has vertices $(0,0),(20,0),(20,16)$ and $(16,0)$ on the coordinate plane. Let $N$ be the number of ways 2 lattice points can be selected on or within the rectangle, such that the midpoint of the segment connecting the selected points is also a lattice point. Compute the remainder when $N$ is divided by 1000. A lattice point is a point in the plane with coordinates $(x, y)$ for integers $x$ and $y$.
3. Jason's favorite function is $f(x)=2^{0}+2^{1}+\ldots+2^{x}$ for any nonnegative real $x$. His second favorite function is $g(x)=\frac{x+1}{2 x^{2}+1}$ for any positive real $x$ less than 2016. Compute the sum of all possible values for $g(f(x))$ that are defined.
4. In parallelogram $A B C D$ let $P$ and $Q$ be on sides $\overline{A B}$ and $\overline{A D}$ respectively, such that $\frac{A Q}{Q D}=\frac{1}{3}$ and $\frac{A P}{P B}=\frac{2}{5}$ Furthermore, define point $E$ on diagonal $B D$ such that $B E=2$ and let $P Q$ meet $A E$ at $T$ and $A C$ at $S$. Given that $B D=6$ compute the ratio $\frac{S T}{P Q}$ as a common fraction.
