TEAM 1 SOLUTION:

If we think about the side lengths of the triangle, we realize that the side lengths of 14, 48, and 50 is actually a pythagorean triple. This is simply a larger 7-24-25 triangle. Therefore, the legs have lengths of 14 and 48 and the hypotenuse has a length of 50. Because the triangle is a right triangle, we can use the formula $b\ x\ h\ /\ 2\ =\ A$ and this gives us 14 * 48 / 2 = 336 cm^2.

TEAM 2 SOLUTION

We know that S = DT, where S is speed, D is total distance, and T is total time. We know the distance between Winchester and Losechester is 120km, and the train makes a round trip, therefore it travels the distance twice, or 240km. Now, we need to find the total time that elapsed during this round trip.

On the departure trip, the train travelled at a speed of 60km/h, so dividing 120 by 60 gives us 2 hours for the departure trip. For the return trip, we divide 120km by the speed of 40km/h and this gives us 3 hours to complete the return trip, totalling 5 hours for both the departure trip and the return trip. The average speed is then 240km divided by 5 hours, or 48km/h.

TEAM 3 SOLUTION

Clarification: the dice are assumed to be 6-sided, numbered with numbers 1 through 6 on each of the 6 faces. In order to find the possible sums, we just need to find the maximum and minimum sums. The maximum sum will be given by the maximum number possible for each die multiplied by the number of dice. This is 6 * 3 = 18. The minimum is much the same, with the minimum possible number for each die multiplied by the number of dice. This is 1 * 3 = 3.

Therefore, we have the total range, being the numbers 3 through 18, inclusive. Therefore the total number of possible sums is 18 - 3 + 1 = 16. We need to make sure to add one at the end to the count because 3 is included in the count.

TEAM 4 SOLUTION

The probability that you draw a red ball is given by (Number of red balls)/(Total number of balls in the bag), and the probability for any given draw is given by 1 - (Number of red balls)/(Total number of balls in the bag). This can also be expressed as (Number of balls that are not red)/(Total, number of balls in the bag).

At the start, there are 10 balls that are not red and 20 balls, giving us the fraction 10/20 for our first draw that we do not get any red balls. After the first draw, there are 9 balls that are not red that are left in the bag and now there are 19 balls that are left in the bag. Our 3 draws can be expressed with fractions as follows: 10/20, 9/19, and 8/18. When we simplify these fractions, we get ½, 9/19, and 4/9. Now, we multiply them all together to get the probability that the three consecutive draws don't have any red balls. Further simplifying, we get 2/19 as our final probability.

TEAM 5 SOLUTOIN

To start, keep in mind that 0 is a whole number. Therefore, the first 98 whole numbers is the numbers 0 through 97. The product of the first 98 whole numbers, includes zero, therefore, the product of the first 98 whole numbers is zero. The sum of the first 98 whole numbers is 0 + 1 + 2 + 3 + 4… + 96 + 97. This can be represented by the formula n*(n+1)/2, where n is the last number in the sequence being summed, in this case, 97.

We don't actually need to calculate exactly what this is, because we only need to calculate the units digit. Plugging in 97 for n, we get 97*98/2, and we would like to find the units digit. The units digit is the digit in the ones place. We can simplify this expression a bit by realizing that 98/2 = 49. Now, this leaves us with 97*49. We only care about the one's place, so we can look at only the ones place for multiplication, or 7 * 9. 7 * 9 = 63, and the one's place of 63 has the value 3. 0 - 3 = -7, so the unit digit is 7.

TEAM 6 SOLUTION

In order to solve this, we need to find the total number of possible to arrange the letters and then eliminate any extra counts due to some of the letters being the same. There are 8 letters in total, so there are 8! Possible ways to arrange them. However, because there are two Ds and two Is, we need to divide 8! By (2! * 2!) to make sure that we aren't double counting. 8! = 8 * 7 * 6 * 5 * 4 * 3 * 2 * 1, or 40320. Dividing 40320 by (2! * 2!), which has a value of 4, gives us 10080 different possible "words" that can be made.

TEAM 7 SOLUTION

The volume of a cube is given by the equation S^3, where S is the side length of the cube. The volume of a cylinder is given by the equation pi * R^2 * H, where R is the radius of the base, and H is the height of the cylinder. The two solids have the same volume as each other, so we can express this as S^3 / (pi * R^2 * H) = 1.

When the side length of the cube is tripled, we have a new volume of $(3S)^3$ or $27S^3$. The radius of the cylinder is doubled, so the new volume of the cylinder is $\pi * (2R)^2 * H$, or $4\pi * R^2 * H$. The volume of the cube was multiplied by 27, and the volume of the cylinder was multiplied by 4, so the new ratio of the volume of the cube to the volume of the cylinder is 27/4. $27 + 4 = 31$.

TEAM 8 SOLUTION

The volume of the initial slab is $2 * 8 * 14$, or 224 cubic inches. The slab is painted red on all sides. In order for a cube to be completely painted on all sides after the slab is cut up and rearranged, the exterior faces of the cube have to be red.

~~A cube has 8 vertices, 12 edges, and 6 faces. The cube will have 8 corner pieces, each of which has 3 faces painted red, as well as a number of edge pieces and center faces. The number of edge pieces, or pieces which have two adjacent faces painted red, can be represented by the expression $12 * (S-2)$, where S is the side length of the new cube. The -2 comes from the fact that the cubes that cover the vertices are already accounted for, and the exist on the end of each edge. Finally, the faces can be represented by the expression $6 * (S-2)^2$. There are 6 faces, and after accounting for the vertices and edges, there is a square in the center with side length S-2.~~

~~For our initial slab, we have 8 vertex cubes, $(6+12)*2$ edge cubes, and the rest are face cubes with one side painted red. This gives us 36 edge cubes, and the remaining $224 - 8 - 36$ cubes are all face cubes. There are 180 face cubes. We also need to account for the fact that inside the final cube there are cubes inside. We do not care whether these cubes have any red faces or not, because they are on the inside of the final cube and thus are not visible. We will need $(S-2)^3$ interior cubes.~~

~~With 36 edge cubes, we can equate this to the number of edge pieces needed, and we get $36 = 12 * (S-2)$. Solving for S, we find that $S = 5$. Now we need to check if we have enough face cubes to make a cube with side length 5 that is completely painted red. We will need $6 * (5-2)^2$ face cubes, or 54 face cubes. We have 180 face cubes, and $180 - 54 = 126$. Finally, we need to check to make sure we have enough interior cubes. $S = 5$, so the number of interior cubes needed is $(5-2)^3 = 27$. $126 > 27$, so we have enough cubes. Therefore, the volume of the largest cube that can be made is S^3 or $5^3 = 125$ cubic inches.~~

Our initial cube will have 8 vertex cubes.
The number of edge cubes can be represented as $(2-2)*4+(8-2)*4+(14-2)*4$, which equals 72.
The remaining cubes are all face cubes, which there are 144 of.

If we have a cube of volume 216, the side length will be 6.

We will need 8 vertex cubes, which we have enough of.
We will need (6-2)*12 edge cubes, which is 48, so we have enough cubes.
We will need (6-2)^2*6 face cubes, which is 96, so we have enough cubes.
Finally, we will need (6-2)^3 interior cubes, which is 64. But do we have enough?

224 - 8 - 48 - 96 = 72, so we have enough interior cubes.
Therefore the volume of the largest cube that can be made is 216 cubic inches.

TEAM 9

First, let's factor 2023. The factors of 2023 are 1, 7, 17, and 2023, because 7 * 17 * 17 = 2023. Therefore, 2023 has 6 factors. This is because we can take the power of the prime factors of 2023, 7^1 and 17^2, add one to them for each of those numbers to the 0th power, and that gives us 2 * 3 = 6 factors.

The third smallest number with 6 factors will be 20.

TEAM 10 SOLUTION:

Natural numbers are as follows: 1, 2, 3, etc...

Prime numbers are only divisible by one and themselves. Such as, 2, 3, 5, 7, 11, 13, 17, 19....

We need to find the number of powerful numbers, numbers that can be expressed in the form x^n where x is a natural number and n is a prime number, between 1 and 514, inclusive.

This means that x^n must be less than 514.

Rather than counting by powers for each number, going 2^2, 2^3, 2^5, it's more helpful to find the different possible prime exponents and finding the maximum value of x for a given n rather than the other way around.

because 1^n is always 1, we don't need to worry about 1^n, but it is counted once. We will never hit a maximum value of n for 1^n.

In order to maximize the value of n, we need to find the smallest value of x that is not 1.

That ends up being 2.

We can find 2^n, trying out various prime numbers, before finding that 2^7 is 128 and 2^11 is 2048.

Therefore, our maximum value of n is going to be 7, with only 1^7 and 2^7 being possible.

Therefore, the possible values for n given x is greater than 1 are going to be 2, 3, 5, and 7.

The largest value of x for n=5 is 3, with 3^5 = 243.

The largest value of x for n=3 is 8, with 8^3 = 512.

The largest value of x for n=2 is 22, with 22^2 = 484.

There are 22 cases for x^2, 8 cases for x^3, 3 cases for x^5, and 2 cases for x^7.

22 + 8 + 3 + 2 = 35 powerful numbers.

However, just as we mentioned earlier for x=1, we need to make sure that we're not double counting.

1 is counted 4 times, so we subtract 3 from our count to make sure that we don't count it extra times.

The only other number that has this problem is 64, which is both 4^3 and 8^2, and also equivalent to 2^6.

Therefore, we must subtract 1 more from our count to make sure that 64 is not double counted.

35 - 3 (accounting for 1) - 1 (accounting for 64) = 31 powerful numbers.

GUTS 1.1

The measures of angles in a triangle sum to 180 degrees. Therefore, 180 = (2x+17) + (x+43) + (5x-48). Combining terms, we get 180 = 8x + 12. Using algebra to solve this equation, we get that x = 21. Solving for the measures of each of the angles substituting 21 for x, we get angle measures of 59, 64, and 57. Therefore, the measure of the largest angle of the triangle is 64.

GUTS 1.2

Using the method to solve for the next hexagonal number, we can compute the next terms, or we can observe and try and find a pattern. Starting with 7, 7 is 6 greater than 1, and the third term, 19, is 12 greater than 7, or 2 * 6 greater than 7. Note that 19 is 18 greater than 1, or 6 * (1 + 2) greater than 1. The second term is 6 greater, the third term is 6 + 12 greater, the fourth term is 6 + 12 + 18 greater, and so on…

For the first 10 hexagonal numbers, we can represent this as

1 * 10 +
6 * 1 * 9 +
6 * 2 * 8 +
6 * 3 * 7 +
6 * 4 * 6 +
6 * 5 * 5 +
6 * 6 * 4 +
6 * 7 * 3 +
6 * 8 * 2 +
6 * 9 * 1

This can be reexpressed as 6 * (9 + 16 + 21 + 24 + 25 + 24 + 21 + 16 + 9). Combining 9 with 21, and 16 with 24, we can simplify this to 6 * 145.

Then, we can express the sum of the first 10 hexagonal numbers as 10 + 6 * 145, which solves to be 880.

GUTS 1.3

If you run back and forth 10 times, then you start and end at the same spot at the end. You have not changed your position, relative to the train. Your speed will increase to 310 km/h and decrease to 290 km/h, but in the end your position relative to the train remains unchanged. Therefore, your average speed relative to the ground will be the same as the train, 300 km/h.

GUTS 2.1

Gabe's first lap of 400 meters will be 60 seconds.
His second will be 61 seconds.
His third will be 62 seconds.
His fourth will be 63 seconds.
He doesn't have to run a 5th lap because 1600/400 = 4.
Therefore, his total time to run is going to be 246 seconds.
There are 60 seconds in a minute, meaning Gabe will run for 4.1 minutes.

GUTS 2.2

If we leave everything in terms of exponents, x^y^z = x^(y*z), so 2^2^2^2 simplifies to 2^16.
x^y * x^z = x^(y+z). So the bottom becomes (2^4)^2, or 2^16.Therefore, the answer is 1.

GUTS 2.3

The volume of a cylinder is represented by the equation V = pi*R^2*H, where R is the radius of the base of the cylinder and H is the height of the cylinder. 50 * 20^2 * pi= 20000pi. 1 cubic centimeter = 1 cubic milliliter, so the answer is 20000pi.

GUTS 3.1

In order for Mr Mac to end up in the exact same place that he started, he has to perfectly undo his actions. If there are 4 actions, then that allows for 2 steps and 2 more steps to undo his first 2 steps. This is because if Mr Mac takes 3 steps away from the start point, all going east, then he can at most take one step west, and he is not going to arrive at the place he started. If mr mac steps north for his first action then south for his second, his third step guarantees that he will leave the starting point and he will have to undo his third step with the fourth step. Therefore, he will always have 2 actions and 2 more to undo his movements. It doesn't matter what the 2 steps that step away from the starting point are, as long as his 2 undo movements are the exact opposite that correspond to the movement. North and south go together, and east and west go together. Only one of each of the four movements undoes the other, so if the first two steps don't matter, then there is a (¼)^2 chance of mr mac going back to the starting location after 4 moves, or 1/16.

GUTS 3.2

There are 20 songs in total, so there are 20 possibilities for the first song, 19 for the second song, 18 for the third song, and so on.
There are 20! Possible song orderings. Each song is different. If the first 9 songs must be ordered in alternating classical and jazz, then there are 5! * 4! Possible arrangements of classical and jazz songs. For the rest of the songs, there are 11! Possible arrangements of songs. Therefore, the possible arrangements given the classical and jazz condition is 5! * 4! * 11!, out of 20! Possible arrangements.
We can express this as $\frac{5! * 4! * 11!}{20!}$ 11! And 20! Will cancel out many terms, and 5! And 4! Will cancel out some more. Eventually, after we finish cancelling, we are left with $2^4 + 3^2 + 5^1 + 7^1 + 11^0 + 13^1 + 17^1 + 19^1$, and 4 + 2 + 1 + 1 + 0 + 1 + 1 + 1 = 11.

GUTS 3.3

If we divide the left equation by a on both sides, we get a + b - 3 = 0, so a + b = 3.
If we cube this equation, we get (a+b)^3 = 3^3 = 27.
Expanding this, we get a^3 + 3a^2b + 3ab^2 + b^3 = 27.

Taking the equation on the right, we can multiply both sides by 2b^2 and we end up with ab^2 = 8 - a^2b.
Moving the variables over to one side, we get ab^2 + a^2b = 8. Now, we multiply both sides by 3 and we get 3a^2b + 3ab^2 = 24.

Now, we can substitute this into our equation earlier, giving us a^3 + 24 + b^3 = 27.
Subtract 24 from both sides, and we get a^3 + b^3 = 3.

GUTS 4.1

If the greatest common denominator of 60 and x is 6, and the least common multiple of 60 and x is 180, we can start by finding the prime factorization of 60 and 180.
Prime factorization of 60 is 2^2 * 3 * 5
Prime factorization of 180 is 2^2 * 3^2 * 5

Because 180 is 3 times 60, and 60 is divisible by 3 and not 9, it means that x is a multiple of 9.
X is also divisible by 2 because the gcd of x and 60 is 6. X is not divisible by 4 or by 5 because that would make the gcd a different number.
Multiply 9 by 2 and you get 18.

GUTS 4.2

50 students in a class. 33 like math, 33 like music, and 33 like recess. 33 + 33 + 33 = 99.
17 students like both math and music, 17 like both music and recess, and 17 like both math and recess. 17 + 17 + 17 = 51.
Now, we can use the equation $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$, putting in each group, and we get 50 = 33 + 33 + 33 - 17 - 17 - 17 + x. Therefore, x = 2.

GUTS 4.3

The sum of the factors includes 1 and itself, so at the very least, for prime numbers, the sum of the factors will be x + 1, where x is the number itself. This means that we can put a maximum on the numbers at 35, because 35 + 1 = 36. However 35 also has factors of 5 and 7, so 35 is not the answer. We can also realize that if a number is even, it must have odd factors because an even number plus an odd number is an even number is an odd number, and 36 is even, and 1 is always included in the sum, so we can rule out any powers of 2. It’s pretty easy to just start checking numbers at this point, because there are relatively few possibilities. Eventually we figure out that the answer is 22.

GUTS 5.1

In base 3, the number 21 equals 3*2 + 1*1 = 7. The rest of the base 10 number is comprised of 100 * x + 60, which can be made of a combination of 9s, 27s, 81s, 243s, and 729s. Conveniently, 243 + 27 = 260, so our base 10 number is 267, with base 3 representation 101021.

GUTS 5.2

Solve each of the terms inside the parentheses, and you get 9, 64, and 441.

Each of these terms is a perfect square, so when you take the square root of the product of these numbers, you multiply their square roots by each other to get the value of a. 3 * 8 * 21 = 504, so a = 504.

GUTS 5.3

Using the equation (a+b)(a-b) = a^2 - b^2, we can make our life much easlier.
Thus, we can simplify the equation to
1 + 4 - 1 + 16 - 4 + 64 - 16 + 256 - 64 + 1024 - 256.

Conveniently, many of the terms cancel each other out and we’re left with 1024.

GUTS 6.1

If we space out the green blocks, then we have 10 green blocks, with an empty space in the middle of the blocks. We have 9 red blocks, and 11 empty spaces (9 spaces in between the green blocks plus the two ends), so we just need to figure out how many ways there are to put 9 blocks into 11 holes.
11! / (9! * 2!) = 5 * 11 = 55

GUTS 6.2

We can set q = 1, r = 1, s = 2, and t = 0 and solve for the expression on the right, giving us an answer of 8.

Alternatively, we can expand the equation on the right, and we are left with q^2s^2 + q^2t^2 + r^2t^2 + r^2s^2, which conveniently happens to be the product of the two other equations.

GUTS 6.3

We know that the three fifths perfect number must be divisible by 5. With a finite number of possibilities, just start checking from the bottom up.
5 doesn’t work
10 doesn’t work
15 works. 1 + 3 + 5 = 9, 9/3*5 = 15

GUTS 7.1

If we solve for T_3, we get that 1 * T_3 = 2^2 - 1 = 3.
If we solve for T_4, we get that 2 * T_4 = 3^2 - 1 = 4.
Continuing on, T_n = n.
Therefore, T_{2023} = 2023.

GUTS 7.2

In order to have a 1 in the units digit when you square a number, your ones place needs to be 1 or 9.
The 4 numbers are: 01, 49, 51, and 99.
They sum to 200.

GUTS 7.3

Use the equation for sum of consecutive cubes, (n(n+1))^2/4, and we get that the sum of the cubes 1 through 10 equals 3025. The square root of 3025 is 55.

GUTS 8.1

The pattern is recognizable, so the next term will be 6^5, or 7776.

GUTS 8.2

If you look at each part within the parentheses, we get (x-1)(x)+(x)(x+1), which just becomes 2x^2.

This continues for the numbers 1 through 20, so we just need twice the sum of the squares of the numbers 1 through 20.
Using the sum of consecutive squares formula: n(n+1)(2n+1)/6, we get the sum of the squares of the numbers 1 through 20 is 2870, and twice that is 5740.

GUTS 8.3

MOTHER 1

There are 6 “Greats” and they are grandparents, so the family tree reaches up 8 generations. In generation 1, there is just my mother, in generation 2, there are my two grandmothers, in gen 3, there are 4… in generation X there will be $2^{(X-1)}$ mothers. $1 + 2 + 4 + 8…. + 2^N = 2^{(N+1)}-1$. The 8th generation will have 2^7 mothers, so in total there are $2^8 -1 = 255$ mothers in total .

MOTHER 2

Let’s start from Percy’s Grandparents. Percy’s grandparents are 3 pure martians and one full human. Two martians will have a pure martian child, and a full human and pure martian will have a 50% martian child. When you have a 50% martian and full martian parent, you get a 75% martian child, so Percy is 75% martian. Now, once you have a 50% martian mom and a 75% martian dad, you average the two and their daughter Marcy will be 62.5% martian.

MOTHER 3

Using algebra, we can set up some equations, but first, let’s define some variables:
We will represent the age of Graham with A
The age of Gretchen will be B
The age of Grace will be C
The age of Griffin will be D

Now we can set up our equations.
$B = 2A$ (Gretchen is twice the age of Graham)
$C = \sqrt{D+2}$
$A = 8D + 2$
$C * D = 21$

We want to solve for the age of Gretchen. To find the age of Gretchen, we need to find the age of Graham. To find the age of Graham, we need to find the age of Griffin. And to find the age of Griffin, we have two equations that equate Griffin’s age and Grace’s age with each other.

Using the equation $C * D = 21$, we can divide both sides by D and get $C = 21 / D$. Now, we can substitute this value for C into the equation $C = \sqrt{D+2}$, giving us $21 / D = \sqrt{D+2}$. In order to remove the square root, we can square both sides, giving us $441 / D^2 = D+2$. We can now multiply both sides by D^2, giving us $441 = D^3 + 2D^2$.

Subtracting 441 from both sides gives us the equation 0 = D^3 + 2D^2 - 441. Using synthetic division, we can find that D-7 is a factor of the polynomial, therefore D = 7 is a solution to the polynomial.

Substituting this into the equation 8D+2 = A, we can figure out that Graham is 58 years old.

Finally, this gives us the answer that Gretchen is 116 years old.

CHESS 1

In a 8 x 8 grid of squares, there are 64 total squares. If each square is white or black, then each square can have 2 possible states, white or black. One square has 2 states, and two squares together would have 4 possible states, being:

Black Black
White White
Black White
White Black

We can see that we have 2^n number of states, where n is the number of squares that we have.

Therefore, if we have 64 total squares, there would be 2^64 total states. To find a, we need to find a if 2^2^a = 2^64. Taking log base 2 of both sides, we get 64 = 2^a. Taking log base 2 again of both sides, we get a = 6.

CHESS 2

There are 16 pieces, 2 of which are rooks. In the first round of replacement, there is a 14/16 chance that Macnus will not replace a rook. In the second round of replacement, there is a 13/15 chance that both rooks have not been replaced with macaroni. This continues on. The expanded expression for probability looks like this:

$\frac{14}{16} * \frac{13}{15} * \frac{12}{14} * \frac{11}{13} * \frac{10}{12}$

If we choose to not simplify any of the fractions while we write the expanded fraction, we can conveniently start cancelling out top and bottom. Finally, we get our simplified fraction as 11/24.

CHESS 3

If there are 6 pawns, 2 knights, and 1 bishop, there are 9 chess pieces in total. If each of these pieces were unique, there would be 9! Combinations. However, because some of these pieces are identical, we have to remove the extra orders that are the same. There are 6! Ways to arrange 6 pawns and 2! Ways to arrange 2 knights. The total number of unique orders is therefore represented as 9!/(6!*2!), which becomes 9*7*4, or 252.

MACMATICS 1

The figure is half of a circle. The shaded region can be thought of as half of a circle, with half of a smaller, inner circle removed. If AB = 8 mm, then the radius of the semicircle is 4. If CD = 4mm, then the radius of the inner semicircle is 2. The area of the semicircle of radius 2 is going to be 2^2 pi / 2, or 2pi mm^2. The area of the semicircle of radius 4 is going to be 4^2 pi / 2, or 8pi mm^2. 8pi - 2pi = 6pi mm^2.

MACMATICS 2

There are 5 types of fish, and we need to consider the minimum to guarantee that Mr. Mac has at least 10 salmon, 15 cod, 13 trout, 18 tuna, or 2023 mackerel.

To solve this, we need to consider the worst case scenario: If mr mac has 9 salmon but then he starts to catch cod, he will have to keep on fishing because he hasn't met the condition yet. We can apply this to each of the fish types: Mr mac can have 9 salmon, 14 cod, 12 trout, 17 tuna, and 2022 mackerel. Then, after Mr Mac catches 1 more fish, he will have satisfied one of the conditions. Therefore, the minimum amount of fish that Mr Mac has to catch is 9 + 14 + 12 + 17 + 2022 + 1 = 2075 fish.

MACMATICS 3

If each identifier has 12 digits in hexadecimal and there are 16 possible digits and no digit is repeated in the Mac address, then our first mac address digit has 16 possibilities, the next has 15, then 14, and so on, as we use up a digit and can no longer use it for that same mac address. This will stop once there are 4 digits unused since each mac address is only 12 digits long. Therefore, the equation is 16 * 15 * 14 … * 6 * 5, which can be represented in the form 16! / 4!. 16 - 4 = 12.

DASH 1 SOLUTION

1 + 2 + 3 … + 10 = 55, and 55/5 = 11, so x = 11.

DASH 2 SOLUTION

Rocky has 4 friends and himself, therefore there are 5 rocks. There are 6 pies, and he shares them evenly amongst his rock friends and himself. Therefore, each rock gets 6/5 pie, or 1.2 pies.

DASH 3 SOLUTION

1 of the 6 sides of a die contains the number 5, therfore there is a ⅙ chance of rolling a 5 on a single die. Because there are two dices, we have to multiply ⅙ by ⅙ to get 1/36 as the chance of rolling two 5s on two separate 6 sided dice. 1 + 36 = 37.

DASH 4 SOLUTION

Using order of operations, we work our way from the inside out. We multiply 3 by 3 first, because multiplication comes first. This gives us 9. 9 + 2 = 11, and 11 * 2 = 22. 22 +1 = 23, and 23 * 1 = 23.

DASH 5 SOLUTION

We know that the area of a square is given by the equation A = S^2, where S is the side length of the square. For simplicity, let us consider two squares, one with area 16 and another with area 1. The side length of the square with area 16 will be 4, and the side length of the square with area 1 will be 1. The perimeter of a square is given by 4S, where S is the side length of the square. The perimeters of the squares will therefore be 16 and 4. 16/4 = 4.

DASH 6 SOLUTION

The factors for 42 are 1, 2, 3, 6, 7, 14, 21, and 42. Of these, 2, 3, and 7 are prime. 2 + 3 + 7 = 12.

DASH 7 SOLUTION

The difference of the digits is zero, So X - 0 = Y, where X and Y represent the values of the digits of the two digit number. Therefore, X = Y, so the two digits have the same value. The sum of the digits is 16, so X + Y = 16. We know that X = Y, so we get that X + X = 16, or that 2X = 16. Solving for X, we get X = 8. The number is therefore 88.

DASH 8 SOLUTION

If we consider the pattern, we can find that the sum of the first term is 1, the first two is -1, the first 3 is 2, the first 4 is -2, the first 5 is 3, the first 6 is -3… and this pattern continues on. If we continue this pattern, we can see that the sum of the first N terms if N is even is equal to -N/2, and the term right before N is equal to N/2. Therefore, the first 2024 terms will have a sum of -1012, while the first 2023 terms will have a sum of 1012.

DASH 9 SOLUTION

Undo the operations to obtain Snorlax's current weight. sqrt(X/2) = 23, so we square 23 and then multiply it by 2. 23^2 = 529 and 529 * 2 = 1058.

DASH 10 SOLUTION

A circle has 360 degrees and a heptagon has 7 sides. The remainder when 360 is divided by 7 is 3.

DASH 11 SOLUTOIN

When a third place runner passes the person in second place, they take second place, not first. The 9th place runner passes 5 people, so she passed 8th, 7th, 6th, 5th, and 4th. Therefore, after this, she is now in 4th place. After this, the 5th place runner passes her, so now she is in 5th place.

DASH 12 SOLUTION

The square numbers are as follows: 1, 4, 9, 16, etc…I have 5 dice in total, having 4, 6, 8, 12, and 20 sides. The minimum value I can get if I sum the value of the dice is 5, because there are

5 dice. The maximum value I can get if I sume the value of the dice is 4 + 6 + 8 + 12 + 20 = 50. Therefore, I can attain all values between 5 and 50, inclusive. The square numbers in the set [5, 20] are 9, 16, 25, 36, and 49, totalling to 5 square numbers.

DASH 13 SOLUTION

The prime factorization of 91 is 7 * 13 , and the prime factorization of 98 is 7 * 7 * 2. Therefore, the smallest positive integer that is divisible by both 91 and 98 is going to be 2 * 7 * 7 * 13. We get this because we have to take the greatest power of each prime number that appears in all prime factorizations. This gives us an answer of 1274.

DASH 14 SOLUTION

1423 * 3145 = 4475335

DASH 15 SOLUTION

In order to find the distance between the vertex of the function and the given point, we need to use the distance formula, which requires that we have the coordinates for the vertex of the function. To find the vertex of the function, we have to remember that the vertex occurs at the x position of x = -b/2a when the equation of the parabola is given in the form y = Ax^2 + Bx + C. B is -6, and A is 3, therefore -b/2a = 1. Now, to get the y position of the vertex, we have to plug in the value we found for x. y = 3(1)^2 - 6(1) + 1, = 3 - 6 + 1 = -2. Therefore, the coordinates for the vertex of the function of y are (1, -2). We can use pythagorean theorem to find the distance between the two points. The horizontal difference is (1- (-1)) = 2, and the vertical distance is (-2 - 1) = 3, so the two legs of our right triangle that we will use to find the distance are 2 and 3. 2^2 + 3^2 = 4 + 9 = 13, so the distance between the two points is sqrt(13).