WINMAC

# Solutions Manual

Winchester Math Competition 2021

## Introduction

This document contains solutions to problems in all 2021 WinMaC rounds: Dash, Theme, Team, and Guts.

It is possible, if not likely, that this document contains errors. While we hope they are limited to typographical ones, there may be mathematical errors as well. If you notice any, please contact us through the problem appeal form.

## Dash Round

1. Using $A, O$, and $K$ to denote apples, oranges, and kiwis, we can write the following equations:
$$3A = 4O(1)$$
$$6O = 2K(2).$$
We are looking for $8K$ in terms of $A$. We can substitute equation (2) into $8K$ to get $8K = 24O$. We then substitute equation (1) into $24O$, getting $8K = \boxed{18}A$.

2. Substituting the expressions for $S$ and $T$ into $2S + T$, we get
$$2(-10x - 8x - 6x - 4x - 2x) + (x + 5x + 9x + 13x + 17x).$$
Simplifying this expression gives $-15x$. We substitute 17 for $x$ to get $\boxed{-255}$.

3. The amount of pizza needed for a single contestant is $\frac{24}{140} = \frac{6}{35}$. Multiplying by 100 gives $\frac{120}{7}$ pizzas. Because we can't order a fraction of a pizza, we bring it up to the whole number above, which is $\boxed{18}$.

4. By inspection, the number of intersection points is maximized when the two parallel lines are coincident with two parallel sides of the rectangle. The number of intersection points is $\boxed{\text{Infinity}}$.

5. Since the last digit of a number determines its parity, we only need to consider the last digit. Because all the possible digits are positive integers, they each are equally likely to be the last digit. 3 out of the 5 possible last digits are odd, so there is a probability $\boxed{\frac{3}{5}}$ that the 5-digit number is odd.

6. 11 is between the denominators of 16 and 6, so $x$ must be between the numerators of 13 and 5. Intuitively, $x$ should be close to but less than 11. Testing a few values gives $x = \boxed{9}$.

7. Assume there are 100 contestants. Before the competition, 40 contestants answered "Yes" and 60 answered "No." $40 \cdot 60\% = 24$ out of 100 contestants were "No" voters who changed their minds. There are currently $40 + 24 = 64$ "Yes" voters, but the problem states that there were only 60 "Yes" voters after the competition, which means 8 "Yes" voters also changed their minds. In total, $24 + 4 = 28$ changed their minds, leaving $\boxed{72}$ out of 100 who didn't change their minds.

8. Note that numbers under 30 cannot have more than 6 factors (this happens to be the number of factors 30 has). We do casework on the number of factors.
2 factors: These numbers must be prime, and there are 10 primes under 30.
3 factors: These numbers must be perfect squares. Excluding 1, there are 4 perfect squares.
Because there is no overlap between these two groups, the final answer is just $10 + 4 = \boxed{14}$.

9. The first fold folds the equilateral triangle in half into a right triangle with area 8. The second fold decreases the size by $\frac{1}{4}$, resulting in a trapezoid with area $\boxed{6}$.

10. We proceed with complementary counting. The number of ways to seat 8 people around a circular table without restrictions is $(8-1)! = 7!$ (This is because we can "fix" one person to account for all rotations). We can group Mark and Kevin into one person to count the ways where they always sit together. There are $6!$ such permutations. $7! - 6! = 6!(7-1) = \boxed{4320}$.

11. Use the second equation $\frac{b}{8} + 33 = 30$ to isolate and solve for $b = -24$. Substituting into the first equation and simplifying gives $a^2 - 6a - 27 = 0$, which factors into $(a-9)(a+3) = 0$. $a = -3$ and $9$ and the possible ordered pairs are $\boxed{(-3,-24) \text{ and } (9,-24)}$.

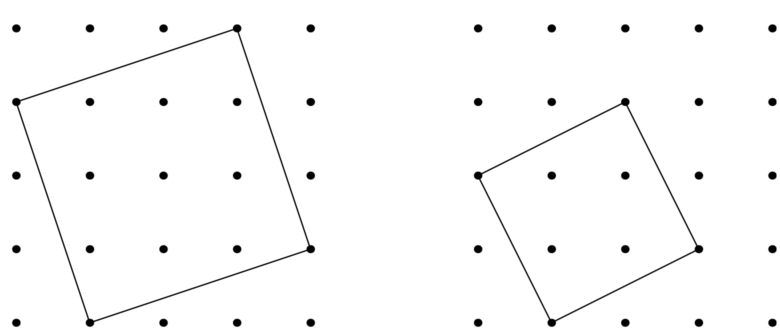
12. Notice that $-2 = -1 \cdot 2$, $10 = 2 \cdot 5$, and in general the $n$-th term is $(n-2)(n+1)$. 340 is $17 \cdot 20$. Thus, the term $a$ is $20 \cdot 23 = \boxed{460}$.

    *Alternative Solution:* Without seeing the pattern $(n-2)(n+1)$, we can first suppose the series can be modeled by a polynomial. Then we can apply the technique of finite difference, where you write down the difference between two adjacent numbers in each row before (for example, the 12 in the second row is $10 - (-2) = 12$).

$$\begin{array}{ccccccccccccc}
-2 & & 10 & & 40 & & 88 & & \ldots & & 340 & & a \\
& 12 & & 30 & & 48 & & & \ldots & & & a-340 & \\
& & 18 & & 18 & & & & \ldots & & & &
\end{array}$$

    After reaching second-order differences, we can see that the difference seems to stay constant at 18, and all rows afterward will become 0. This means that the polynomial is of degree 2. Assuming that all second-order differences are 18, we can backtrack our steps to deduce the entire series: $-2, 10, 40, 88, 143, 238, 340, \boxed{460}$. Note that because we are given more than 3 numbers, there is at most one unique second-degree polynomial that describes the series in the problem statement.

13. Start by counting rectangles with sides parallel to the grid. Each rectangle is formed of 2 vertices from a row matched with 2 vertices from a column. We multiply the ways to choose 2 points from 5 in a row and the ways to choose 2 points from 5 in a column. $\binom{5}{2} \cdot \binom{5}{2} = 10 \cdot 10 = 100$ rectangles.
    Some diagonal rectangles can be made of smaller diagonal squares (with side length $\sqrt{2}$). There are 9 small diagonal squares, and 11 that are a combination of them. Additionally, the following configurations contribute 2 and 8 squares, respectively. There are $9+11+2+8$ diagonal rectangles, amounting to a total of $\boxed{130}$.



14. The first friend can either have or not have each of the 5 cards, hence there are two options for each card and there are $2^5 = \boxed{32}$ total hands. Note that it does not matter how many friends there are nor what the probabilities of each hand are.

15. 54, 145, and 28 have the same remainder when divided by $a$, so the pairwise differences between the them must be divisible by $a$. The differences are 91, 117, and 26. Their only common divisor is 13, so $a = 13$. Using the original numbers we can find that $b = 2$. The answer is $13 \cdot 2 = \boxed{26}$.

# Theme Round

### Jars and Boxes

1. There are 12 apples in total which doesn't change, so there must be greater than $\frac{12}{2} = 6$ red apples for the probability to be greater than $\frac{1}{2}$. You can get 7 red and 5 blue by switching $\boxed{2}$ apples.

2. We are looking for the least common multiple of 5, 4, 12, and 15 which are the denominators of the fractions, which is $\boxed{60}$.

3. By symmetry, the probability that the first bean is green is $\boxed{\dfrac{1}{2}}$.

### Ornithology

1. The number of ways to choose a 4 member team from 14 members is $\binom{14}{4} = \frac{14!}{(14-4)! \cdot 4!} = \boxed{1001}$.

2. If we assume that all birds are Glossy Ibises, then there would be $3 \cdot 11 = 33$ feet of wings. We need $45 - 33 = 12$ more feet. The difference between the wingspans of the Glossy Ibis and Great Blue Heron is 3 feet, so there are $\frac{12}{3} = \boxed{4}$ Great Blue Herons.

   *Alternative Solution:* Set Glossy Ibises as $a$ and Great Blue Heron as $b$, and set up the following equations:

   $$a + b = 11$$

   $$3a + 6b = 45.$$

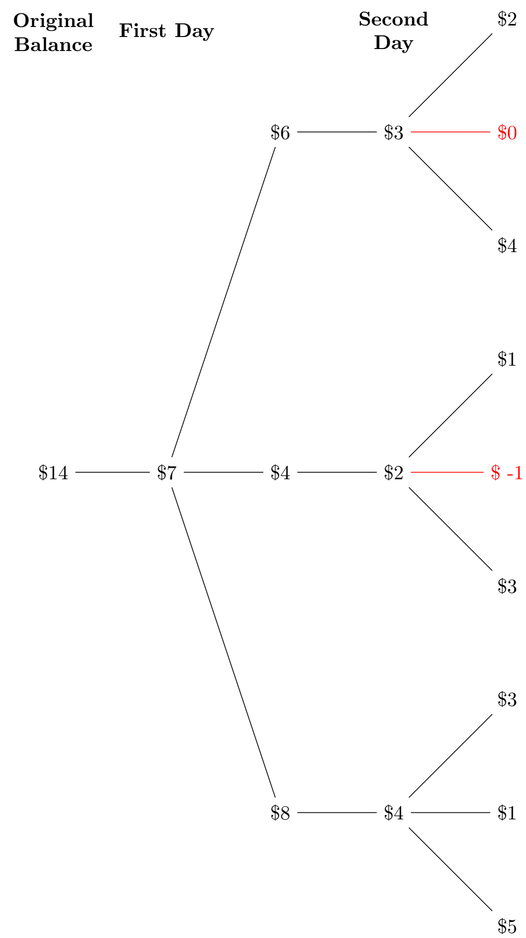
   Solving gives $b = \boxed{4}$.

3. The law of contraposition states that a conditional statement is true if and only if its contrapositive is true. Thus, the "if" clause must negate the "then" clause in the given statement." The "then" clause must negate the "if" clause in the given statement. The only valid option out of the choices is $\boxed{A}$: The bird sees no predator, but the bird gives a warning call.

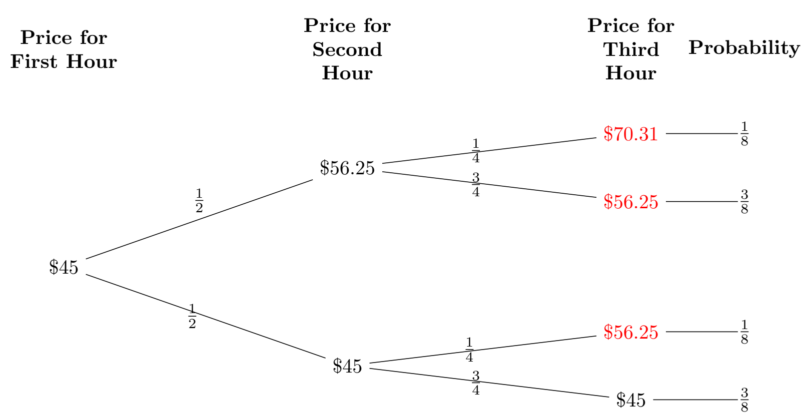
### Willa's Economic Escapades

1. To catch the Bottlefish, Willa needs to fish for $1500 \div 3 = 500$ days. Every day, she earns 200 dollars but loses $0.8 * 200 + 50 = 210$ dollars on costs, leaving her with \$10 less than the day before. However, on the last day, when she finds the fish, any new revenue or costs on bait and necessities are not important. Therefore, she only loses \$10 for 490 days. To have enough money for this, she must start with $490 * 10 = \boxed{\$4900}$.

2. On the first day, Willa spends 50% of the \$14, leaving \$7 in the bank. After, it is equally likely for her to then have \$6 (lose \$1), \$4 (waste \$3), or \$8 (make back \$1). The second day, she spends 50% again and the account has an equal chance of having \$3, \$2, \$4, respectively, left. Each of these outcomes also have equal chances of -1, -3, or +1. Listing out all nine possibilities, we see that only in the outcome of wasting \$3 in the first and second cases would Willa be bankrupt. Hence, the probability that the account is bankrupt after two days is $\boxed{\dfrac{2}{9}}$.



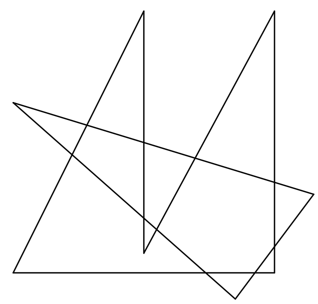

3. The first hour there is a 100% chance that someone bids, so the price increases to $\$36 \cdot 1.25 = \$45$. The probability for the next hour is halved, so there is a 50% chance, or $\frac{1}{2}$ chance, that the price increases to $56.25 and a $\frac{1}{2}$ chance the price remains at $45. The third hour, the probability someone bids halves to $\frac{1}{4}$. Knowing this information, we can create a tree diagram to represent the prices during each hour of the auction, and multiply the probability at each hour to obtain the probability for each final price. In order to make a profit, the final price must be greater than $50, as highlighted in red. Adding up the probabilities, we get $\frac{1}{8} + \frac{3}{8} + \frac{1}{8} = \boxed{\frac{5}{8}}$.



## Team Round

1. There are 6 possibilities for the number $a$, which are 0,1,2,3,4,5, and the same 6 possibilities for $b$. This means there are $6 * 6 = 36$ total possible pairs for $a$ and $b$. Of these 36 pairs, 10 of them give us the difference of 1 that we need. We can find these pairs by going through each value of $a$ and counting up how many values of $b$ would possible for the condition. In the end, this gives us a final probability of $\frac{10}{36} = \boxed{\frac{5}{18}}$.

2. The maximum number of intersection points between a triangle and pentagon, no sides coinciding, is $\boxed{10}$. An example is shown below. [1]



3. Other than just bashing out pairs of $x$ and $y$, we can realize that both sides of the equation must be divisible by 9. This then means that $y$ must be a multiple of 9 for this to hold true. From there, we quickly get $y = 9$ to minimize our value for x, and $x = \boxed{8}$.

4. Every two years, July's temperature changes by $2 - 3.5 = -1.5$ degrees and December's temperature changes by $-2 + 5 = 3$ degrees. Every two years the difference between the temperatures get closer by $1.5 + 3 = 4.5$ degrees, so it takes $2 \cdot \frac{80-44}{4.5} = 16$ years for the temperatures to be equal. In the 17th year, July's temperature increases while December's decreases, so the 18th year is when December's is higher. The answer is $2018 + 18 = \boxed{2036}$.

5. Divisibility for 4 is tested for by checking if the number formed by the last two place values of an integer (the tens and ones place) is divisible by 4. Because of this, we only have to concern ourselves with counting how many numbers from 0 to 99 satisfy the condition. We get 18 after counting, and now we can just multiply this number by the number of possible hundreds places. In the given range, and subtracting the case where it is 4, we get 8, so $18 * 8 = \boxed{144}$.

6. A die can roll a value between 1 and 6 inclusive. Because the number of dice is 4, the largest value on any die that could still satisfy the condition is 3. Otherwise the value would go over, since the other 3 dice have a minimum value of 1 each. So, we break the problem in 3 cases: one where the maximum value on a die is 3, one where the max is 2, and final one where the max is 1. The case for 3 gives 4 possibilities, the case for 2 gives 6, and the case for 1 has no possibilities. $4 + 6 = \boxed{10}$.

7. The information given to us can be expressed in terms of equations where $c$ is the price of a pencil, $p$ is the price of a pen, and $e$ is the price of an eraser. We get

$$7c + 6p + 5e = 5$$

$$13c + 14p + 15e = 15$$

and we want to find what $c + p + e$ equals to find Christian's money. While at first it seems like there isn't enough information, if we add the two equations together we get

$$20c + 20p + 20e = 20.$$

and dividing both sides by 20 gives us $c + p + e = \boxed{1}$.

[1] By the way, if you want to learn more about polygon intersections, see this link: https://arxiv.org/pdf/2002.05680.pdf

8. The terms $a, b, c, d$ are digits in the three digit numbers. Because of this, this means that the values for each one can only be from 0 to 9, inclusive. But another condition in the problem specifies that the values have to be positive, so as a result our range is 1 to 9 inclusive. We can also express the numbers in expanded notation like so
$$100a + 10b + c = abc$$
$$100a + 10a + c = aac$$
$$100c + 10c + d = ccd$$
Added together, we get that
$$210a + 10b + 112c + d = 1111$$
With this knowledge, we can work through our range of possibilities by making cases for $a$ and seeing what numbers would fit it. Keeping in mind that the values must all be distinct, we get that the only values that work would be $a = 2, b = 1, c = 6, d = 9$, so our answer is $\boxed{9}$

9. By far the simplest way to solve this problem is just to bash out all the numbers from 1 to 20. While this might seem intimidating at first, the process can made far faster with a simple shortcut. If in the sequence we find a number we've already seen in a previous sequence, we know that it must lead to the same result. For example, we have the sequence for 1 being
$$1, -1, -2, -3, -3...$$
The sequence for 2 starts as
$$2, 5, 1$$
But because we know that 1 doesn't lead to 0 from the sequence we calculated earlier, we can stop right there can conclude neither does 2 since the whole process is deterministic. Using this logic on all the numbers gives us $3, 4, 9, 10, 11, 12$ for a final tally of $\boxed{6}$

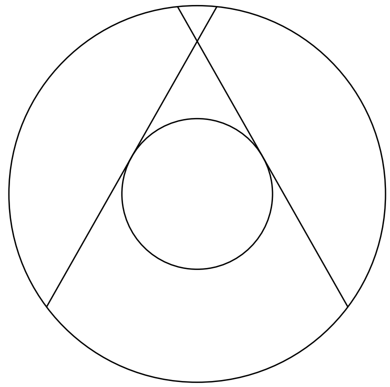
10. We expand the entire polynomial by FOILing so we get[2]
$$x^3 + (-a - b - c)x^2 + (ab + bc + ac)x + (-abc) = x^3 - 12x^2 + 10x - 9$$
From this we obtain
$$a + b + c = 12$$
$$ab + bc + ac = 10$$
$$abc = 19$$
While this doesn't look very useful, once we distribute and expand the other expression we get
$$(a - 1)(b - 1)(c - 1) = abc - (ab + bc + ac) + (a + b + c) - 1$$
And notice from here that we already know the values of the terms, so we just plug in the values to get
$$(a - 1)(b - 1)(c - 1) = 19 - 10 + 12 - 1 = \boxed{20}$$

# Guts Round

### Set 1

1. The donut can be divided into a maximum of $\boxed{5}$ regions, as shown below.

[2]Note that this process is much neater with Vieta's Formulas if you are interested in learning more!



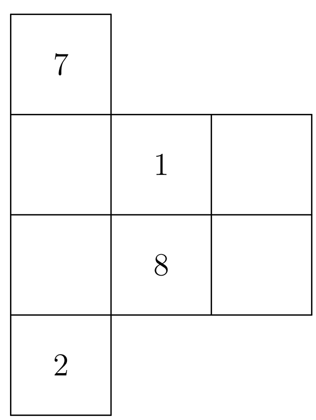
2. Notice that $(x - 82)$ is in the numerator, and everything times $82 - 82 = 0$ will equal 0. Before concluding that the answer is 0, we must also check that the denominator is not 0. The denominator cannot be 0 because all numbers added to $x$ are positive. Hence, the answer is $\boxed{0}$.

3. It takes $\boxed{7}$ days. On day 7, the pond is half covered, and the lily pads double to cover the whole pond for day 8.

**Set 2**

1. The easiest way to solve the problem is testing various cases, with the intuition that dividing by 2 will reduce the number fastest. We want to create factors of 2, so we add 1 to 23, getting 24. The next turns are three divisions by 2 and one minus 3. The minimum turns needed is $\boxed{5}$ $(+1, \div 2, \div 2, \div 2,$ and $-3)$.

2. There are 2 center boxes that share sides or corners with 6 other boxes. Since 1 and 8 are the only numbers not adjacent to 6 other numbers, they must go in those boxes (as shown below). Thus, 2 and 7 are fixed.




Of the remaining 4 numbers, 3 and 5 must be adjacent and 4 and 6 must be adjacent. There are two ways to fill the column with 4 boxes: 7352 and 7462. For each choice of the first column, there are two orientations for the 2 numbers in the third column. Additionally, the entire shape can be reflected horizontally. In total there are $2 \cdot 2 \cdot 2 = \boxed{8}$ ways to fill the boxes.

3. Letting the total cost be $x$, Jasmine pays $0.4x$, Jessica pays 80, and Juliet pays $0.5(0.4x + 80)$. The sum of these expressions must be $x$, allowing us to set a equation in terms of $x$ and to solve for this unknown. You need not use an equation to solve this problem, but this our most straightforward way.

$$0.4x + 80 + 0.5(0.4x + 80) = x$$

$$0.4x + 80 + 0.2x + 40 = x$$

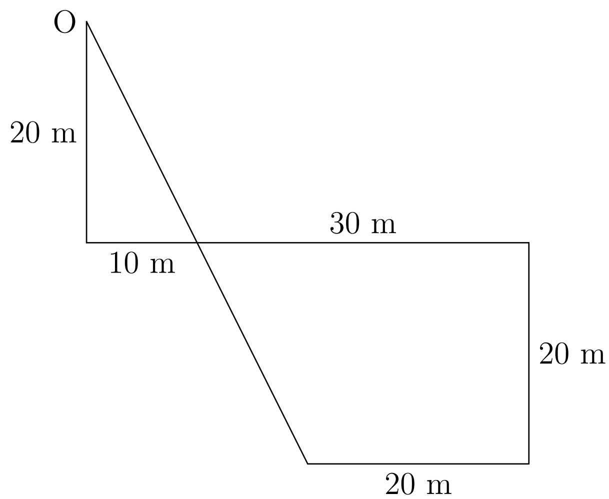
$$0.6x + 120 = x$$

$$120 = 0.4x$$

$$x = \boxed{300}$$

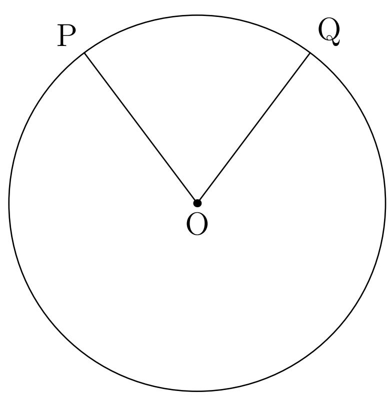
**Set 3**

1. The diagram below shows Yaozu's path, consisting of a triangle and a trapezoid. The area of the triangle is $20 * 10 * 0.5 = 100$ square meters, while the area of the trapezoid is $0.5(30 + 20) * 20 = 500$ square meters. In total, the area of the entire park is $\boxed{600}$ square meters.




2. We are working with the numbers between 1 and 1000 inclusive. The fact that its factors are all even means that such a number can only be a power of 2. If any other prime factor was included, some factors would be odd. The second and fourth conditions mean that after ignoring 1, which is a factor of every number, the number of factors for the number has to be even. This means that only square numbers can satisfy this condition, because they have an odd number of factors before ignoring 1. Any other number has an even number due to factors coming in pairs. This means that Jasmine can only write down $2^2$, $2^4$, $2^6$, and $2^8$, for a total of $\boxed{4}$.

3. Triangle $PQO$ is an equilateral triangle due to the side lengths all being the same. With this knowledge, we can use the fact that any vertex angle in the triangle must be $\boxed{60}$ degrees for our answer.



**Set 4**

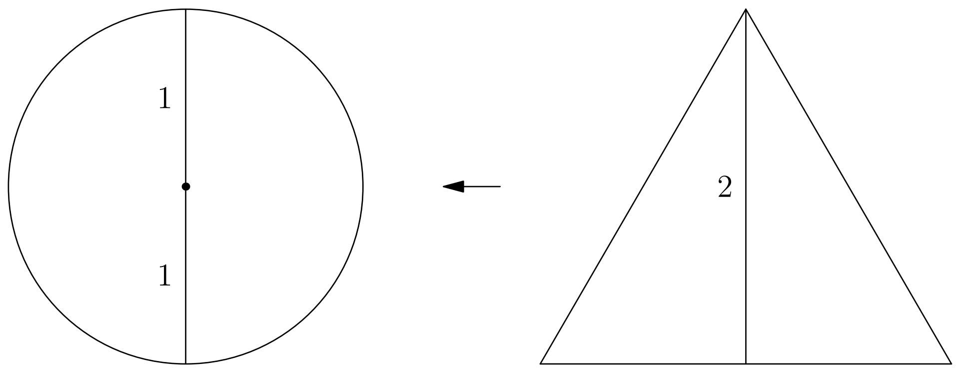
1. Set up a system of equations using the information given. If we set $e$ as Elysia's current age, and $h$ as Hubert's current age, we get the equations

$$e + 4 = h$$

$$\frac{3}{4}(e + 32) + 15 = h + 32$$

   Solving these equations gives us $e = 12$ and $h = 16$, for a final product of $\boxed{192}$.

2. The key here is to think in 3-D. As seen in the diagram, a triangle with a height equal to the diameter of the hoop can just BARELY fit through the circle. Since the diameter is equal to twice the radius, that gives us an answer of $\boxed{2}$.




3. The structure of any phone number that Milo tries is just a string of 1's followed by a string of 2's with a total length of 9 (for example, 111112222). This means that there are 10 possibilities for Milo's mother's phone number ranging from a number with 0 1's and 9 2's to 9 1's and 0 2's. In the worst case scenario Milo guesses $\boxed{9}$ different wrong numbers times before getting the correct one,

**Set 5**

1. Sarah has a total of $6 \cdot 4 = 24$ combinations of socks and pants. Now, we subtract specific combinations that are impossible. First, she can't wear pink socks with blue pants. Also, she can't wear black socks with any color pants other than red, giving us a total of $1+3 = 4$ cases that will not happen. Therefore, her choices are limited to $24 - 4 = \boxed{20}$ outfits.

2. The probability of landing on heads, P(H), and the probability of landing on tails, P(T), are $\frac{2}{3}$ and $\frac{1}{3}$, respectively. (P(H) = 2·P(T) and P(H) + P(T) = 1.) There are three cases in which we get exactly two heads: THH, HTH, HHT. The probability for each case is equal to $\frac{1}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{4}{27}$. Adding up the probability for each case, we have $\frac{4}{27} + \frac{4}{27} + \frac{4}{27} = \frac{12}{27} = \frac{4}{9}$. $4 + 9 = \boxed{13}$.

3. To finish the race after Robert, Jonathan travels 150 meters in 25 seconds. Using the relation $distance = rate * time$, Jonathan's speed must be 6 m/s, and he completes the entire race in 100 seconds. This means Robert finishes the race in 75 seconds, and Gavin finishes in 120 seconds. Using the equation, we figure out that Gavin's speed is $600/120 = 5$ m/s. Gavin must travel for 45 seconds after Robert crosses the finish line. Since his rate is constant, he has $45 * 5 = \boxed{225}$ meters left.

**Set 6**

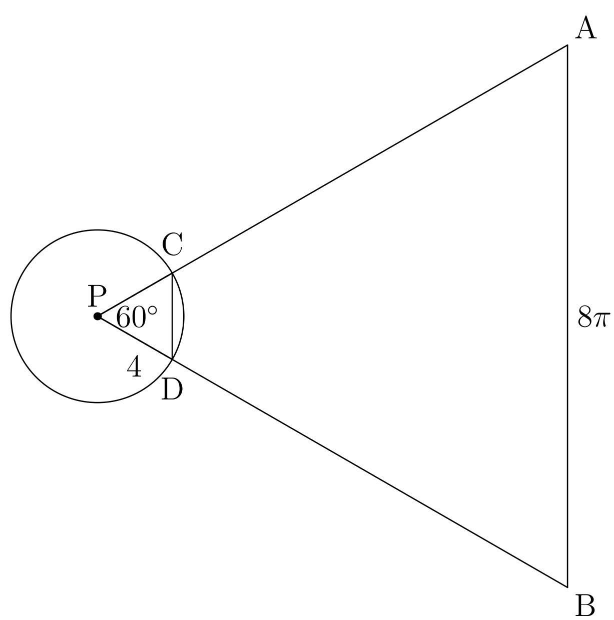
1. The basic intuition for maximizing surface area with a set volume is by making the side lengths as lopsided as possible. In this way the entire volume is channeled into a very stretched prism with high surface area. With this idea, we find that the dimensions that would work best in this case would be a 1 by 1 by 72 rectangular prism with the restriction of the lengths being integers. We then solve for surface area as $2(1 * 1) + 2(1 * 72) + 2(1 * 72)$ to get $\boxed{290}$.

2. Because Aarzu folds the circle over once, effectively every change is doubled. The starting area is $36\pi$ from the standard circle area formula. The cuts in total remove 30 degrees from the semicircle, so 60 degrees from the entire circle. Because in total there are 360 degrees, this leads to $\frac{1}{6}$ of the circle getting cut off, for a current area of $30\pi$. Finally, Aarzu cuts out 5 circles with an area of $\pi$ each. This is once again doubled from the effect of being folded, so $10\pi$ is subtracted, for a final area of $20\pi$ and a final answer of $\boxed{20}$.

3. Since there's a high number of salamanders to start, they will be chosen for many days in a row. We can reduce the amount by four to simplify numbers, giving us 5 bears, 3 dragons, and 6 salamanders left to work with. Following the given behaviors and trying some numbers, we can see that there will eventually be a day where the numbers of each kind are the same (ex. 3 bears, 3 dragons, 3 salamanders, and then 2-2-2). Following this pattern, we will eventually hit 1 of each kind. The janitor will add 3 to the bears, and Rosie will play for the bears until they run out, so the answer is $\boxed{1}$ for bears.

**Set 7**

1. Observe the figure below. We can see that $\angle CPD$ is also equal to $60°$, and $\Delta CPD$ is an equilateral triangle. With $\overline{AB}$ and $\overline{CD}$ parallel, $\angle CDP$ is equal to $\angle ABP$, and we can conclude $\Delta ABP$ is also equilateral. After calculating the length of $\overline{AB}$ as $8\pi$ by the formula for the circumference, this means $\overline{BP}$ is also $8\pi$, giving us our answer of $\boxed{8}$.



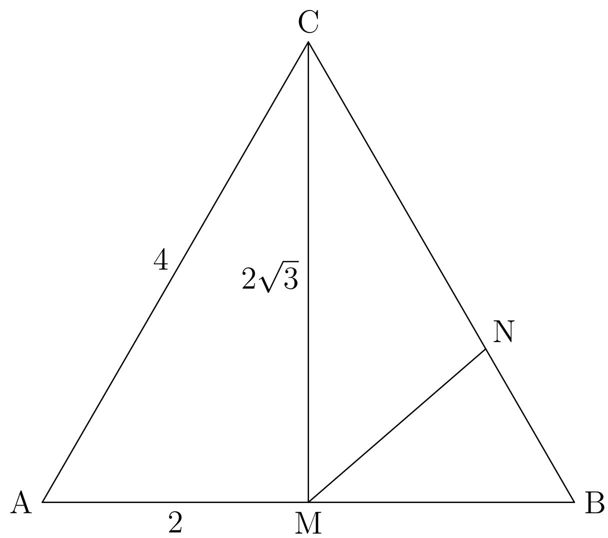

2. First notice that with 4 cards in a hand, and 4 suits, there is only one possibility which is that one card is spades, one is clubs, the next is hearts, and the final one being diamonds. Because of that, the number of possibilities is entirely defined by the numerical/face value since the suit possibilities are set in stone. For the first card, there are 13 choices out of the 13 total. But for the second card, there are only 12 choices due to one choice already being taken up by the first card. Continuing this pattern, this leads to a total of $13 * 12 * 11 * 10 = 17160$ possibilities, for a final answer of $\boxed{15}$ for the digit sum.

3. We can attempt to count all the configurations by separating them into four cases. With no 2 by 1 tiles, there is only 1 configuration: all 1 by 1 tiles that are non-distinguishable. Given one 2 by 1 tile, there are 7 ways to place the tile on the 2 by 3 board. All other squares will be filled in by 1 by 1 tiles. With two 2 by 1 tiles, there are 7 ways again. And the last case, three 2 by 1 tiles, there are only 3 configurations. This gives us a total of $\boxed{22}$ configurations.

**Set 8**

1. It's not hard to divide out the number and get the answer that way, but there's a quicker way if you're observant. Notice that 99999919 is a difference of squares, being equivalent to $100000000 - 81$, otherwise known as $10000^2 - 9^2$. By the difference of squares formula, we also know that $10000^2 - 9^2 = (10000 - 9)(10000 + 9)$. 10009 is one of the factors, so then we know that $m = 10000 - 9 = 9991$, which gets our digit sum of $\boxed{28}$.

2. Consider the ratio of the areas of $\Delta MBC$ to $\Delta ABC$. We know that $\overline{MB}$ and $\overline{AB}$ have a ratio of $\frac{1}{2}$ with the given info. If we take them as the bases of the two triangles, it's clear that $\Delta MBC$ and $\Delta ABC$ will have an AREA ratio of $\frac{1}{2}$ as well because they share a common height. The same logic can be applied to the triangle pair $\Delta MBN$ and $\Delta MBC$ for a ratio of $\frac{1}{3}$ from the base ratio of $\overline{BN}$ and $\overline{BC}$. All together means that the ratio of $\Delta MBN$ to $\Delta ABC$ is $\frac{1}{6}$. The area of the equilateral triangle $\Delta ABC$ can be found through a formula or splitting down the middle, in the end giving us an area of $4\sqrt{3}$. This means that $\Delta MBN$ has an area of $\frac{2\sqrt{3}}{3}$ from our ratio, giving us a final answer of $\boxed{8}$.




3. Consider how Gavin will use all of toilet paper $A$ before using $B$. Once all of $A$ is used, Gavin will use a little bit of $B$, the empty roll $A$ will be replaced before the next use, and Gavin will go back to using $A$ before $B$ runs out. Gavin can use $A$ five times ($\frac{19}{100} * 5 = \frac{95}{100}$, which is just under 1 complete roll), with $\frac{5}{100}$ of $A$ left over for his next trip. Then, he would use the remaining $\frac{5}{100}$ of $A$ as well as $\frac{14}{100}$ of $B$. This would occur every time $A$ runs out, so let's consider that to be one cycle of six uses. Gavin uses $\frac{14}{100}$ of $B$ each cycle, which means a maximum of 7 complete cycles will occur before there is not enough in $B$ to compensate for $A$ running out. On the seventh cycle, Gavin will have used $\frac{14}{100} * 7 = \frac{98}{100}$ of $B$, leaving $\frac{2}{100}$. This is not enough for a complete eighth cycle. However, Gavin can still use $A$ until it runs out, which amounts to 5 times. Adding all of this together gives $6 * 7 + 5 = \boxed{47}$ bathroom trips.