WINMAC

Name: _____ Score: _____ / 160

PLEASE DO NOT FILL IN ABOVE! (the "SCORE" blank)

Grade: _____ Team: _____

This is a round consisting of 10 challenging problems to be done in 30 minutes. You may communicate and discuss problems with people on your team. Problems are in roughly ascending difficulty, and each problem is worth 16 points. Any figures in the test may not be to scale.

No aids are permitted aside from pencils, pens, and provided scratch paper. In particular, no calculators or other computers are permitted. Communication with other people on your own team is allowed.

Record your answers in the box corresponding to the correct problem. Only answers printed in the boxes below will be scored.

Your Answers

1.	3.	5.	7.	9.
2.	4.	6.	8.	10.

1. Let a, b be integers such that $0 \leq a \leq 5$ and $0 \leq b \leq 5$. What is the probability that their difference is 1? Your answer should be a simplified fraction.

2. What is the maximum number of intersections possible between a triangle and a pentagon? Assume that no sides coincide (no sides are shared).

3. If x and y are positive integers such that $11y - 9x = 27$, then what is the smallest possible value of x?

4. In 2018, the average temperatures for July and December in Boston were 80 and 44 degrees Fahrenheit, respectively. Due to climate change, imagine that the average temperature in July increases by 2 degrees and falls 3.5 degrees in alternating years. While in December, temperatures decrease by 2 degrees and rise by 5 degrees in alternating years. So, in 2019, the average temperatures for July and December in Boston were 82 and 42 degrees, respectively. In what year will the temperature in December be higher than the temperature in July?

5. How many three-digit numbers are divisible by 4 but do not contain the number 4?

6. I have 4 different colored 6 sided dice. How many ways can I get a sum of 6?

7. Albert, Bud, and Christian are buying stationary at the Muji store. Albert only has three five-dollar bills, while Christian only has one-dollar bills. The store gives no change, so the boys can only use the bills that they have. It cost 5 dollars for Bud to buy 7 pencils, 6 pens, and 5 erasers, and Albert can buy a maximum of 13 pencils, 14 pens, and 15 erasers. If Christian buys one pencil, one pen, one eraser, and uses all of his one-dollar bills, how much money did he have to begin with?

8. Let a, b, c, d, be distinct, positive integers, where abc, aac, and ccd are three 3-digit numbers that add up to 1111. Find the sum of all possible solutions for d.

9. I have a function $f(x)$ that repeats itself by putting its output back in as an input until it reaches 0.

$$\text{If } x \text{ is even, } f(x) = 2x + 1$$

$$\text{If } x \text{ is odd, } f(x) = \frac{x-3}{2}$$

For example, if I start with $x = 1$. Then the sequence goes

$$1 \rightarrow -1 \rightarrow -2 \rightarrow -3 \rightarrow -3...$$

How many numbers from 1 to 20 will reach 0 when put into this function?

10. Suppose that a, b, and c are values such that no matter what x is, we have

$$(x - a)(x - b)(x - c) = x^3 - 12x^2 + 10x - 19.$$

What is the value of $(a - 1)(b - 1)(c - 1)$?